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A detailed derivation of extended Jones matrix representation for twisted nematic liquid crystal displays

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A detailed derivation of the extended Jones matrix representation for twisted nematic liquid crystal displays at oblique incidence (which was previously published by the author) is given. It is shown that this representation reduces to the well-known ordinary Jones matrix representation at normal incidence.

1. Introduction

To calculate the optical transmission of twisted nematic liquid crystal displays (TN-LCDs), the Jones matrix method [1] is generally used for normal incidence and 4×4 matrix method [2, 3] is commonly employed for oblique incidence. Because Jones matrix is a 2×2 matrix, it is more easily understood and applied than the 4×4 matrix method. Several authors [4–9] have attempted to generalize the Jones matrix to the oblique incidence case. In our previous papers [6, 7], a simple expression of the extended Jones matrix for the TN-LCD was given. We used this Jones extended matrix representation to calculate the optical transmissions for various LCD systems which included TN, super-twist nematic (STN), parallel electrically control birefringent (ECB) and homeotropic ECB. The results were compared with those obtained by the faster 4×4 matrix method [3] with a spectrum average. In general, results obtained by these two methods agreed reasonably well. Due to the fact that this representation is expressed explicitly in terms of LC and cell parameters, it is straightforward to apply our extended Jones matrix representation to optical simulations of TN-LCDs. However, it has been pointed out [9] that this representation is complicated and not intuitive. Thus, in this paper, we give a detailed derivation of this representation which was not able to be done in our previous publications [6, 7] due to space limitations. We also show that this representation reduces to the ordinary Jones matrix at normal incidence. The derivation of the extended Jones matrix is given in § 2. Correction of transmission losses in two air-to-polarizer interfaces is discussed in § 3. Reduction of the extended Jones matrix to the ordinary Jones matrix at normal incidence is discussed in § 4. The conclusion is given in § 5.

2. Formulation of extended Jones matrix for TN-LCDs

Consider a plane wave incident at an oblique angle on the surface of a TN-LCD as shown in the figure. Without losing generality, we can always choose an orthogonal xyz coordinate system such that $\mathbf{k}$ lies on the x–z plane and the xy plane is parallel to the glass substrate surface. Also, the direction of the $+z$ axis is pointing from the entrance polarizer to the exit polarizer. Thus we have

$$\mathbf{k} = k_0(\sin\theta_k,\ 0,\ \cos\theta_k), \tag{1}$$

where θ_k is polar angle of $\mathbf{k}$, $k_0 = \omega/c = 2\pi/\lambda$, and λ is the wavelength of the incident light in free space. The entire LCD system can be divided into N layers. The first layer is the entrance polarizer and the last layer is the exit polarizer. The second and the $(N-1)^{\text{th}}$ layers are glass substrates. The LC layer is approximated by a stack of $N-4$ uniaxial homogeneous layers. Each of these N layers can be characterized by a dielectric tensor:

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \tag{2}$$

where [10]

$$\varepsilon_{xx} = n_o^2 + (n_e^2 - n_o^2)\cos^2\theta\cos^2\phi, \tag{3}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = (n_e^2 - n_o^2)\cos^2\theta\sin\phi\cos\phi, \tag{4}$$

$$\varepsilon_{xz} = \varepsilon_{zx} = (n_e^2 - n_o^2)\sin\theta\cos\theta\cos\phi, \tag{5}$$

$$\varepsilon_{yy} = n_o^2 + (n_e^2 - n_o^2)\cos^2\theta\sin^2\phi, \tag{6}$$

$$\varepsilon_{yz} = \varepsilon_{zy} = (n_e^2 - n_o^2)\sin\theta\cos\theta\sin\phi, \tag{7}$$

$$\varepsilon_{zz} = n_o^2 + (n_e^2 - n_o^2)\sin^2\theta, \tag{8}$$

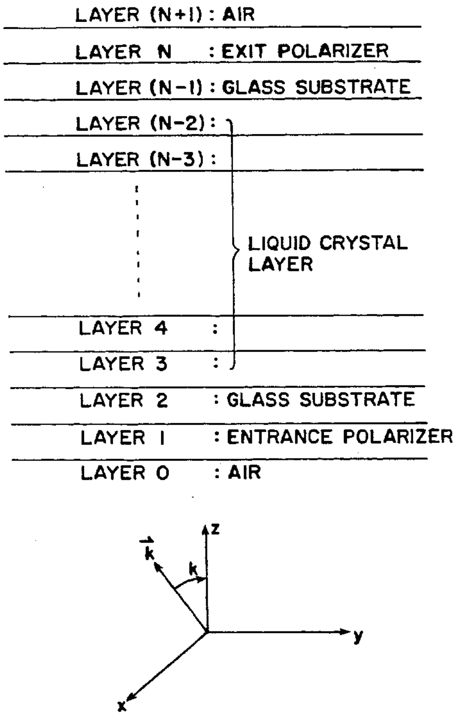

Figure. Schematic diagram of a TN-LCD, which is divided into N layers. The propagation vector $\mathbf{k}$ lies on the xz plane of an orthogonal xyz coordinate system, in which the xy plane is parallel to the glass substrate surface.

Here, n_o and n_e are, respectively, the ordinary and extraordinary indices of refraction of the liquid crystal medium or polarizer layer. For liquid crystal layers, θ is the tilt angle of the LC director (i.e. the angle between the LC director and the x–y plane) and ϕ is the angle between projection of the LC director on the xy plane and the x axis. For polarizer layers, n_o and n_e are complex numbers, θ is the angle between the extraordinary optical axis (absorption axis) of the polarizer layer and the xy plane, and ϕ is the angle between projection of the extraordinary optical axis of the polarizer layer on the xy plane and the x axis. For the usual polarizer arrangement, θ is zero. For glass substrates, $n_e = n_o$ and ε reduces to a diagonal matrix.

For each layer, assuming the plane wave

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \exp(i(\mathbf{k}\cdot\mathbf{r} - \omega t)) = \mathbf{E}_0 \exp(i(k_x x + k_z z - \omega t)), \tag{9}$$

and

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}_0 \exp(i(\mathbf{k}\cdot\mathbf{r} - \omega t)) = \mathbf{H}_0 \exp(i(k_x x + k_z z - \omega t)) \tag{10}$$

is a solution of Maxwell's equations in the non-magnetic medium characterized by a dielectric tensor of equation (2), we obtain

$$\frac{\mathbf{k}}{\mathbf{k}_0} \times \left(\frac{\mathbf{k}}{\mathbf{k}_0} \times \mathbf{E}_0\right) + \boldsymbol{\varepsilon}\cdot\mathbf{E}_0 = 0, \tag{11}$$

and

$$\mathbf{H}_0 = \frac{\mathbf{k}}{\mathbf{k}_0} \times \mathbf{E}_0. \tag{12}$$

Substituting $\mathbf{k} = (k_x, 0, k_z)$ and $\mathbf{E}_0 = (E_{x0}, E_{y0}, E_{z0})$ into equation (11), we obtain

$$\left[-\left(\frac{k_z}{k_0}\right)^2 + \varepsilon_{xx}\right]E_{x0} + \varepsilon_{xy}E_{y0} + \left(\frac{k_x}{k_0}\frac{k_z}{k_0} + \varepsilon_{xz}\right)E_{z0} = 0, \tag{13}$$

$$\varepsilon_{yx}E_{x0} + \left[-\left(\frac{k_z}{k_0}\right)^2 - \left(\frac{k_x}{k_0}\right)^2 + \varepsilon_{yy}\right]E_{y0} + \varepsilon_{yz}E_{z0} = 0, \tag{14}$$

$$\left(\frac{k_x}{k_0}\frac{k_z}{k_0} + \varepsilon_{zx}\right)E_{x0} + \varepsilon_{zy}E_{y0} + \left[-\left(\frac{k_x}{k_0}\right)^2 + \varepsilon_{zz}\right]E_{z0} = 0. \tag{15}$$

Here $k_x = k_0 \sin\theta_k$. Equations (13) to (15) have a non-trivial solution when

$$\begin{vmatrix} -\left(\frac{k_z}{k_0}\right)^2 + \varepsilon_{xx} & \varepsilon_{xy} & \frac{k_x}{k_0}\frac{k_z}{k_0} + \varepsilon_{xz} \\ \varepsilon_{yx} & -\left(\frac{k_z}{k_0}\right)^2 - \left(\frac{k_x}{k_0}\right)^2 + \varepsilon_{yy} & \varepsilon_{yz} \\ \frac{k_x}{k_0}\frac{k_z}{k_0} + \varepsilon_{zx} & \varepsilon_{zy} & -\left(\frac{k_x}{k_0}\right)^2 + \varepsilon_{zz} \end{vmatrix} = 0. \tag{16}$$

For the dielectric tensor specified by equations (3) to (8), the solutions [11] of equation (16) are

$$\frac{k_{z1}}{k_0} = \left(\left[n_o^2 - \left(\frac{k_x}{k_0}\right)^2\right]\right)^{1/2} \tag{17}$$

$$\frac{k_{z2}}{k_0} = -\frac{\varepsilon_{xz}}{\varepsilon_{zz}}\frac{k_x}{k_0} + \frac{n_o n_e}{\varepsilon_{zz}} \times \left(\left[\varepsilon_{zz} - \left(1 - \frac{n_e^2 - n_o^2}{n_e^2}\cos^2\theta\sin^2\phi\right)\left(\frac{k_x}{k_0}\right)^2\right]\right)^{1/2} \tag{18}$$

$$\frac{k_{z3}}{k_0} = -\left(\left[n_o^2 - \left(\frac{k_x}{k_0}\right)^2\right]\right)^{1/2} \tag{19}$$

$$\frac{k_{z4}}{k_0} = -\frac{\varepsilon_{xz}}{\varepsilon_{zz}}\frac{k_x}{k_0} - \frac{n_o n_e}{\varepsilon_{zz}} \times \left(\left[\varepsilon_{zz} - \left(1 - \frac{n_e^2 - n_o^2}{n_e^2} \cos^2\theta \sin^2\phi \right) \left(\frac{k_x}{k_0} \right)^2 \right] \right)^{1/2}. \tag{20}$$

This implies there are four eigen-waves propagating in the medium. Two eigen-waves corresponding to k_{z1} and k_{z2} propagate in the $+z$ direction and are the transmitted waves; while the other two corresponding to k_{z3} and k_{z4} propagate in the $-z$ direction and are the reflected waves. For each k_{zi}, there is a corresponding electric field $(E_{xi}, E_{yi}, E_{zi}) = (E_{xi0}, E_{yi0}, E_{zi0}) \exp(i(k_x x + k_{zi} z - \omega t))$, where $(E_{xi0}, E_{yi0}, E_{zi0})$ must satisfy equations (13) to (15). Due to the condition given in equation (16), only two out of these three equations (equations (13) to (15)) are linearly independent. Therefore, we can express two components of the electric field in terms of the third component. Because reflected waves inside a TN-LCD are usually small, in the following treatment we assume they can be neglected. In other words, we assume only two transmitted eigen-waves 1 and 2 are propagating in the medium. For eigen-wave 1, we express E_{y1} in terms of E_{x1}. By using equations (14) and (15), we obtain

$$E_{y1} = e_{y1} \frac{E_{x1}}{e_{x1}} = e_{y1} M_{x1}, \tag{21}$$

where

$$e_{x1} = \left[\left(\frac{k_{z1}}{k_0} \right)^2 + \left(\frac{k_x}{k_0} \right)^2 - \varepsilon_{yy} \right] \left[\left(\frac{k_x}{k_0} \right)^2 - \varepsilon_{zz} \right] - \varepsilon_{yz}\varepsilon_{zy}, \tag{22}$$

$$e_{y1} = \left[\left(\frac{k_x}{k_0} \right)^2 - \varepsilon_{zz} \right] \varepsilon_{yx} + \left(\frac{k_x}{k_0}\frac{k_{z1}}{k_0} + \varepsilon_{zx} \right) \varepsilon_{yz}, \tag{23}$$

and

$$M_{x1} = \frac{E_{x1}}{e_{x1}} = \frac{E_{x10}}{e_{x1}} \exp(i(k_x x + k_{z1} z - \omega t)). \tag{24}$$

Similarly, for eigen-wave 2, we express E_{x2} in terms of E_{y2}. By using equations (13) and (15), we obtain

$$E_{x2} = e_{x2} \frac{E_{y2}}{e_{y2}} = e_{x2} M_{y2}, \tag{25}$$

where

$$e_{x2} = \left[-\left(\frac{k_x}{k_0} \right)^2 + \varepsilon_{zz} \right] \varepsilon_{xy} - \left(\frac{k_x}{k_0}\frac{k_{z2}}{k_0} + \varepsilon_{xz} \right) \varepsilon_{zy}, \tag{26}$$

$$e_{y2} = \left[-\left(\frac{k_{z2}}{k_0} \right)^2 + \varepsilon_{xx} \right] \left[\left(\frac{k_x}{k_0} \right)^2 - \varepsilon_{zz} \right] + \left(\frac{k_x}{k_0}\frac{k_{z2}}{k_0} + \varepsilon_{zx} \right) \left(\frac{k_x}{k_0}\frac{k_{z2}}{k_0} + \varepsilon_{xz} \right), \tag{27}$$

and

$$M_{y2} = \frac{E_{y2}}{e_{y2}} = \frac{E_{y20}}{e_{y2}} \exp(i(k_x x + k_{zi} z - \omega t)). \tag{28}$$

Using equations (21), (24), (25) and (28), the **E**-vector (formed by tangential components of the total electric field, **E**) at any point can be expressed in terms of mode vector

$$\begin{bmatrix} M_{x1} \\ M_{y2} \end{bmatrix}$$

as

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{x1} \\ E_{y1} \end{bmatrix} + \begin{bmatrix} E_{x2} \\ E_{y2} \end{bmatrix} = \boldsymbol{S} \begin{bmatrix} M_{x1} \\ M_{y2} \end{bmatrix}, \tag{29}$$

where

$$\boldsymbol{S} = \begin{bmatrix} e_{x1} & e_{x2} \\ e_{y1} & e_{y2} \end{bmatrix}, \tag{30}$$

Using equations (24) and (28), the mode vector propagates from the bottom of n^{th} layer to the top of the n^{th} layer by

$$\begin{bmatrix} M_{x1} \\ M_{y2} \end{bmatrix}_{n,d_n} = \boldsymbol{G}_n \begin{bmatrix} M_{x1} \\ M_{y2} \end{bmatrix}_{n,0}, \tag{31}$$

where

$$\boldsymbol{G}_n = \begin{bmatrix} \exp(ik_{z1}d) & 0 \\ 0 & \exp(ik_{z2}d) \end{bmatrix}_n, \tag{32}$$

and d_n is the thickness of the n^{th} layer.

Using equations (29) and (31), the **E**-vector at the bottom of the n^{th} layer is related to the **E**-vector at the top of the n^{th} layer by

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_{n,d_n} = \boldsymbol{J}_n \begin{bmatrix} E_x \\ E_y \end{bmatrix}_{n,0}, \tag{33}$$

where

$$\boldsymbol{J}_n = \boldsymbol{S}_n \boldsymbol{G}_n \boldsymbol{S}_n^{-1}. \tag{34}$$

Using the boundary condition that tangential components of the **E**-field are continuous at each layer interface, i.e.

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_{n+1,0} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}_{n,d_n}, \tag{35}$$

we obtain

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_{n+1} = \boldsymbol{J}_n \begin{bmatrix} E_x \\ E_y \end{bmatrix}_n . \tag{36}$$

Here simplified notations

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_n = \begin{bmatrix} E_x \\ E_y \end{bmatrix}_{n,0} \tag{37}$$

and

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_{n+1} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}_{n+1,0} \tag{38}$$

are used.

Equation (34) is the extended Jones matrix representation at an oblique incidence for the n^{th} layer. Equations (34) and (36) can be understood as follows. $\boldsymbol{S}_n^{-1}$ transforms the **E**-vector at the bottom of the n^{th} layer into the mode vector. $\boldsymbol{G}_n$ then propagates the mode vector from the bottom of the n^{th} layer to the top of the n^{th} layer. Finally, $\boldsymbol{S}_n$ transforms the mode vector at the top of the n^{th} layer back into the **E**-vector at the top of the n^{th} layer, which is equal to the **E**-vector at the bottom of the $(n+1)^{\text{th}}$ layer (from the boundary condition). Note that due to the special form of $\boldsymbol{G}_n$, $\boldsymbol{J}_n$ can equivalently be rewritten as

$$\boldsymbol{J}_n = \begin{bmatrix} 1 & c_2 \\ c_1 & 1 \end{bmatrix}_n \boldsymbol{G}_n \begin{bmatrix} 1 & c_2 \\ c_1 & 1 \end{bmatrix}_n^{-1} \tag{39}$$

where $c_1 = e_{y1}/e_{x1}$ and $c_2 = e_{x2}/e_{y2}$.

Applying equation (36) to every layer, the matrix equation that related the transmitted **E**-vector and the incident **E**-vector is given by

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_{N+1} = \boldsymbol{J} \begin{bmatrix} E_x \\ E_y \end{bmatrix}_0 , \tag{40}$$

where

$$\boldsymbol{J} = \boldsymbol{J}_N \boldsymbol{J}_{N-1}, \ldots, \boldsymbol{J}_2 \boldsymbol{J}_1 \tag{41}$$

is the extended Jones matrix representation for the entire LCD system at oblique incidence. The total optical transmission t_{op} is given as

$$t_{\text{op}} = \frac{|E_{x,N+1}|^2 + \cos^2(\theta_{\text{p}})|E_{y,N+1}|^2}{|E_{x,0}|^2 + \cos^2(\theta_{\text{p}})|E_{y,0}|^2}, \tag{42}$$

where

$$\theta_{\text{p}} = \sin^{-1}(\sin(\theta_k)/Re(n_{\text{p}})), \tag{43}$$

in which $Re(n_{\text{p}})$ stands for the average of the real parts of the two indices of refraction (n_{e} and n_{o}) of the polarizer.

3. Correction of transmission losses in the air-LCD interfaces

The transmission losses in the entrance and exit surfaces of an LCD cell are usually significant, but are neglected in the above extended Jones matrix formulation. Two methods can be used to take care of these transmission losses. The first method is to take into account these transmission losses using the optical modification procedure, which was published in our previous paper [6]. The second method is to modify the extended Jones matrix of equation (41) to take care of transmission losses in two air-to-polarizer interfaces. In the second method, $\boldsymbol{J}$ in equation (41) is replaced by

$$\boldsymbol{J}' = \boldsymbol{J}_{\text{Ext}} \boldsymbol{J} \boldsymbol{J}_{\text{Ent}} = \boldsymbol{J}_N \boldsymbol{J}_{N-1}, \ldots, \boldsymbol{J}_2 \boldsymbol{J}_1 \boldsymbol{J}_{\text{Ent}}, \tag{44}$$

where [12]

$$\boldsymbol{J}_{\text{Ent}} = \begin{bmatrix} \dfrac{2\cos\theta_{\text{p}}}{\cos\theta_{\text{p}} + n_{\text{p}}\cos\theta_k} & 0 \\ 0 & \dfrac{2\cos\theta_k}{\cos\theta_k + n_{\text{p}}\cos\theta_{\text{p}}} \end{bmatrix} \tag{45}$$

and

$$\boldsymbol{J}_{\text{Ext}} = \begin{bmatrix} \dfrac{2n_{\text{p}}\cos\theta_k}{\cos\theta_{\text{p}} + n_{\text{p}}\cos\theta_k} & 0 \\ 0 & \dfrac{2n_{\text{p}}\cos\theta_{\text{p}}}{\cos\theta_k + n_{\text{p}}\cos\theta_{\text{p}}} \end{bmatrix} \tag{46}$$

The total transmission is calculated by equation (42) with θ_{p} being replaced by θ_k.

4. The Jones matrix representation at normal incidence

For normal incidence, we have

$$c_1 = -\frac{\cos\phi}{\sin\phi}, \tag{47}$$

$$c_2 = \frac{\cos\phi}{\sin\phi}, \tag{48}$$

and

$$\boldsymbol{G} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} \exp\left(i\dfrac{2\pi}{\lambda} n_{\text{eeff}} d\right) & 0 \\ 0 & \exp\left(i\dfrac{2\pi}{\lambda} n_{\text{o}} d\right) \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \tag{49}$$

where

$$n_{\mathrm{eeff}} = \frac{n_{\mathrm{o}} n_{\mathrm{e}}}{(n_{\mathrm{o}}^2 + (n_{\mathrm{e}}^2 - n_{\mathrm{o}}^2)\sin^2\theta)^{1/2}}. \qquad (50)$$

By using equations (47) to (49), equation (39) reduces to

$$\boldsymbol{J}_n = \exp\left(i\frac{\pi}{\lambda}(n_{\mathrm{eeff}} + n_{\mathrm{o}})_n d_n\right)\begin{bmatrix}\cos(\phi_n) & -\sin(\phi_n)\\ \sin(\phi_n) & \cos(\phi_n)\end{bmatrix} \times \begin{bmatrix}\exp\left(i\frac{\pi}{\lambda}(n_{\mathrm{eeff}} - n_{\mathrm{o}})_n d_n\right) & 0\\ 0 & \exp\left(-i\frac{\pi}{\lambda}(n_{\mathrm{eeff}} - n_{\mathrm{o}})_n d_n\right)\end{bmatrix} \times \begin{bmatrix}\cos(\phi_n) & \sin(\phi_n)\\ -\sin(\phi_n) & \cos(\phi_n)\end{bmatrix} \qquad (51)$$

at normal incidence.

Equation (51) is the ordinary Jones matrix representation obtained by Chandrasekhar at normal incidence [13], except a complex phase factor which is omitted in Chandrasekhar's expression. This phase factor can be neglected for a non-absorptive medium. But it is important when the ordinary Jones matrix is used to represent a polarizer whose indices of refraction are characterized by complex numbers, since without it the ordinary Jones matrix will not give a correct exponential decay for both ordinary and extraordinary waves propagating inside the polarizer.

5. Conclusions

We have given a detailed derivation of the extended Jones matrix representation for TN-LCDs. The derivation was based on the assumptions that reflected waves inside the TN-LCD system could be neglected. The boundary condition of the electric field was employed to connect the extended Jones matrix of every adjacent layer to form the extended Jones matrix of the entire LCD system. To account for significant transmission losses occurring in two air-to-polarizer interfaces, a modification of the extended Jones matrix was made. We also show that this representation reduced to the ordinary Jones matrix at normal incidence. The extended Jones matrix representation for each layer (equation (34)) and that for the entire LCD system (equations (41) and (44)) can easily be understood and applied. Comparisons of the optical transmissions obtained by this extended Jones matrix method and those obtained by the faster 4×4 matrix method with a spectrum average are not given here since detailed comparisons have already been carried out for various LCD systems in references [6] and [7]. Furthermore, generalizations of the extended Jones matrix formulation to the biaxial medium is under investigation [14].

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